Safety, Stability, and Efficiency of Taxi Rides

Martin Aleksandrov[1(✉)] and Tobias Labarta[2]

[1] Freie Universität Berlin, Berlin, Germany
martin.aleksandrov@fu-berlin.de
[2] Technische Universität Berlin, Berlin, Germany
labarta@campus.tu-berlin.de

Abstract. We propose a *novel* approach for limiting possible sexual harassment during taxi rides, where penalizing harassing drivers and matching them to passengers play key roles. In this paper, we focus on the matching part. In particular, we propose a *novel* two-sided market model, with drivers on one side and passengers on another side, where drivers have (1) *safety* preferences, (2) *profit* preferences, and (3) *gender* preferences, for passengers, and passengers have (1) *safety* preferences, (2) *delay* preferences, and (3) *gender* preferences, for drivers. Given these three-layer preferences, we study increasing the safety and stability in matchings, thus possibly reducing the chance of sexual harassment. In addition, we combine safety and stability with maximizing total profit or minimizing total delay. We design a number of algorithms throughout the paper and measure their safety, stability, and efficiency.

Keywords: Social choice · Multi-layer preferences · Taxi dispatching

1 Introduction

Sexual harassment could occur at various places during our daily routine: at home, on the way to work, at work, and on the way home: see e.g. a study by the US Center for Disease Control and Prevention (CDC) [1]. Fighting sexual harassment in public transport is particularly challenging because sexual harassment there is often committed by strangers whose personal information we may never be able to collect for further processing in court. For example, one Statista study[1] from 2016 shows that 18+-year-old women across France are often subject to the following sexual harassment types at public transport: out of 6227 respondents, 83% go for "whistling", 87% go for "invasive presence", 36% go for "intrusive question", 36% go for "insult, threat", 41% go for "sexual exhibition", 40% go for "sexual assault", 1% goes for "rape", and 4% go for "other". By these statistics, it follows that some of these victims have experienced multiple sexual harassment types either during different rides or during the same ride, possibly even by the same perpetrators.

[1] https://www.statista.com/statistics/1118528/types-bullying-women-transport-public-france/.

N. Moniz et al. (Eds.): EPIA 2023, LNAI 14116, pp. 401–412, 2023.
https://doi.org/10.1007/978-3-031-49011-8_32

Victims of public transport sexual harassment may therefore prefer private transportation such as taxi services over the perceived threat of taking a metro or a bus. Although taxis are considered safer, there has been an increasing number of sexual harassment cases in them as well [4]. This is especially concerning because taxis might be the only option for women from underrepresented groups such as those with disabilities and those from various cultural minorities. But, in cases where this option can no longer be considered safe, women from such groups might feel less socially included, which in turn could harm their status in society. This could apply to women with low incomes, who already face restrictions in single-sex ride-sharing services, where they cannot share a trip with male passengers [2]. But, *how can we reduce sexual harassment in taxis?*.

Without having a reliable way of tracking harassment behavior, we cannot even hope to have a reliable way of fighting such behavior. Hence, tracking harassment behavior is necessary for fighting it. Currently, the existing approach for fighting harassment works as follows: (1) match drivers and passengers; (2) collect harassment claims; (3) issue fines to drivers. We depict it on the left side of Fig. 1. The drawback of this approach is that taxi companies remain unaware of and, therefore, cannot track the harassment behavior of their drivers. In response, we propose the following novel approach for tracking harassment behavior: (1) collect sexual harassment claims, (2) penalize harassing drivers, and (3) match drivers and passengers. We depict it on the right side of Fig. 1.

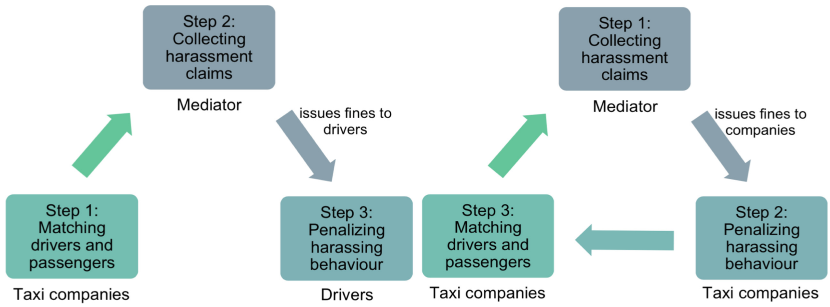

Fig. 1. Two approaches for fighting sexual harassment during taxi rides: (left) the existing approach; (right) the proposed approach.

1.1 Step 1: Collecting Harassment Claims

We propose to involve an independent governance body that can serve as a mediator between passengers and drivers. For example, the Taxi and Limousine Commission (TLC) in New York opened an Office of Inclusion, where passengers can submit sexual harassment claims after rides requested via an app, over the phone, or from the street. Such a body could thus regularly (e.g. every month) be sending to taxi companies **anonymous** feedback about how many claims were submitted against each of their drivers, without revealing any passenger data, preventing leakages of it to drivers.

1.2 Step 2: Penalizing Harassing Drivers

We propose the following novel **disciplinary** mechanism: the mediator (e.g. TLC) could issue fines to taxi companies, based on claims from the last feedback cycle (e.g. last month), thus motivating companies to design local prevention mechanisms; then, companies could detain the permits of harassing drivers and ask them to attend mandatory subsidized workshops before receiving their permits back. Companies could thus deploy a disciplinary approach for educating staff about basic principles for preventing sexual harassment during rides in their local areas, such as avoiding cheesy language, explaining victim perspectives, and increasing cultural awareness.

1.3 Step 3: Matching Drivers and Passengers

Most cases of sexual harassment are done by male drivers to female passengers [4]. This is possible partly because companies do *not* elicit from passengers their preferences over the gender of their potential matching driver. We propose to elicit such preferences during bookings. Also, we let passengers submit to dispatchers the times they can depart from their pickup locations, allowing dispatchers to compute the *delays* of drivers for picking up passengers. Furthermore, based on past rides, some passengers and drivers might have made, say through familiarity, *safe* rides and, for future matches, prefer such rides. We propose to elicit the permit numbers of familiar drivers during bookings. In this paper, we thus propose a two-phase matching method for **reducing possibly the chance of submitting sexual harassment claims to the mediator, by firstly maximizing the level of safety (Phase 1) and secondly optimizing the stability over the gender types (Phase 2)**.

2 Related Work

Taxi dispatching relates to the seminal work of Karp et al. [8]. Since then, there are various extensions of this work. Mehta [11] presents a thorough survey of such works. Unlike us, the vast majority of these works consider one layer of preferences. For example, Zhao et al. [13] studied stable matchings under distance preferences of passengers over drivers and identical profit preferences of drivers over passengers. Also, Lesmana et al. [10] investigated efficient and fair matchings under delay preferences of passengers over drivers. Indeed, multi-layer matchings received significantly less attention in the research literature. One exception is the work of Chen et al. [3] who looked at a model where agents have multiple preference lists. Thus, they studied one notion of two-layer stability and showed that matchings that satisfy two-layer stability may not exist. We extend this work by running extensive simulations to confirm how close to two-layer stability we might get in practice. Furthermore, we are not aware of any prior work that focuses on integrating a theoretical two-sided market model with three-layer preferences into a practical approach for reducing possible sexual harassment in taxis.

3 Our Novel Model

We let t denote a point in time. We consider n drivers from $D = \{d_1, \dots, d_n\}$ that are available at time t. We also consider m passengers from $P = \{p_1, \dots, p_m\}$ whose requests for rides have not been serviced by time t. As our model is a two-sided market model, we refer to the parameter $\max\{n, m\}$ as *market size*. Pick driver $d_i \in D$. In practice, drivers service requests one after another. Thus, as in [13], we aim at matching $\min\{n, m\}$ drivers, each servicing exactly one request. We suppose that they have a depot location l_{d_i}. This might be a central or current location. Pick passenger $p_j \in P$. We let p_j have a pickup location l_{p_i}. This might be a public or private location.

3.1 Preferences Layers

Layer 1: Safety Preferences: Based on familiarities (e.g. location experiences, driver-passenger friendships), we let passengers and drivers submit incomplete safety preference lists. For passengers, we encode these as a complete safety relation $S_1 = (s_{1ji})_{m \times n}$, where $s_{1ji} = 1$ if p_j feels safe to be serviced by d_i, and else $s_{1ji} = 0$, indicating that it is unknown to the central planner whether p_j feels safe to be serviced by d_i. Similarly, for drivers, we encode these as a complete safety relation $S_2 = (s_{2ij})_{n \times m}$, where $s_{2ij} = 1$ if d_i feels safe to service the area of location l_{p_j}, and else $s_{2ij} = 0$, indicating that it is unknown to the central planner whether d_i feels safe to service the area of location l_{p_j}. We thus let $\Sigma = (\sigma_{ji})_{m \times n}$ denote the *joint safety* relation, where $\sigma_{ji} = 1$ if $s_{1ji} = 1$ and $s_{2ij} = 1$ hold, and else $\sigma_{ji} = 0$. We thus let passengers and drivers prefer rides that are safe for both of them to rides that feel unsafe for any of them.

Layer 2: Profit-delay Preferences: We suppose that each d_i derives a *profit* $\pi_{ij} > 0$ from servicing passenger p_j. The profit π_{ij} might also indicate how profitable p_j is for the business if serviced by d_i. For example, π_{ij} could be the charge for the ride minus the fuel cost. We let $\Pi = (\pi_{ij})_{n \times m}$. At the We suppose that t_{ij} denotes the estimated time of arrival (ETA) of d_i for picking up p_j at l_{p_j} from l_{d_i}. We let t_{ij} be computed with respect to (WRT) time t when we want to match drivers and passengers. That is, we suppose that $t_{ij} \geq t$ holds. We next suppose that p_j knows the time τ_j after which they have to depart from l_{p_j}. As p_j is not serviced by time t, we let $\tau_j \geq t$ hold. If p_j is not picked up by τ_j then they experience a *delay*, defined as follows: $\delta_{ji} = 0$ if p_j is matched to d_i and $t_{ij} \leq \tau_j$; $\delta_{ji} = (t_{ij} - \tau_j) > 0$ if p_j is matched to d_i and $t_{ij} > \tau_j$; $\delta_{ji} = 0$ if p_j is matched to d_k for some $k \neq i$. When booking a ride, p_j may not have knowledge of the location of d_i, which means that it is not possible for them to calculate t_{ij}. For this reason, we assume that p_j submits τ_j to a central planner who can calculate each t_{ij} and each δ_{ji}. We let $\Delta = (\delta_{ji})_{m \times n}$.

Layer 3: Gender Preferences: Let us assume that the central planner has access to the gender of their drivers. We let $\mathcal{G} = \{g_f = \text{female}, g_m = \text{male}, g_d = \text{diverse}\}$ contains the gender types. We let each p_j have a gender ranking that they submit to the central planner. For example, p_j may prefer ($\succ$) female drivers to male drivers and male drivers to diverse drivers. In this case, their gender ranking is $g_f \succ g_m \succ g_d$. We suppose that rankings may contain ties. For example, p_j may be indifferent ($\sim$) between male drivers and diverse drivers. In this case, their gender ranking is $g_f \succ g_m \sim g_d$. We also suppose that this ranking may be incomplete. For example, p_j may submit that they prefer female drivers. In this case, their gender ranking is g_f. We suppose that the central planner can complete (arbitrarily) incomplete rankings by appending the missing gender types and assuming passengers are indifferent among them. For example, they can complete g_f to $g_f \succ g_m \sim g_d$. By using the gender ranking for each p_j, we can calculate a complete ordinal ranking over the drivers, $\phi_j := d_{i_1} \circ \ldots \circ d_{i_n}$ with $\circ\{\succ, \sim, \succeq\}$ and where $(i_1, \ldots, i_n)$ is a permutation of $(1, \ldots, n)$, which for g_i of d_i and g_h of d_h is defined as follows: $d_i \succ d_h$ iff $g_i \succ g_h$; $d_i \sim d_h$ iff $g_i \sim g_h$; $d_i \succeq d_h$ iff $g_i \succ g_h$ or $g_i \sim g_h$. Additionally, we let each d_i have a gender ranking that the central planner could use to calculate similarly a complete ordinal ranking over the passengers, $\chi_i := p_{j_1} \circ \ldots \circ p_{j_m}$ with $\circ\{\succ, \sim, \succeq\}$ and where $(j_1, \ldots, j_m)$ is a permutation of $(1, \ldots, m)$, but we assume that they do not know this gender ranking as, otherwise, they would have had to elicit the gender data from passengers, which may discourage them from participation.

3.2 Matchings

Eligibility graphs: We consider a bipartite graph $G = (V_P, V_D, E)$, where the sets of passenger and driver vertices are V_P and V_D, respectively, and the *eligibility* relation between them is $E \subseteq V_P \times V_D$. For example, when going to the airport, d_i may not be eligible for servicing p_j in case p_j has many luggage items that cannot fit in the vehicle trunk of d_i. Thus, for each $(p_j, d_i) \notin E$, we set $\pi_{ij} = 0$ and $\delta_{ji} = K >> \max_{(p_g,d_h)\in E} \delta_{gh}$.

Matchings: We write $M = \{(p_{j_1}, d_{i_1}), \ldots, (p_{j_{\min\{n,m\}}}, d_{i_{\min\{n,m\}}})\}$ for a *matching* in G, where $j_1, \ldots, j_{\min\{n,m\}}$ are different indices among $1, \ldots, m$ and $i_1, \ldots, i_{\min\{n,m\}}$ are different indices among $1, \ldots, n$.

Jointly safe matchings: The overall joint safety level in M is $TS(M) = \sum_{(p_j,d_i)\in M\cap E} \sigma_{ji}$. We say that matching M is *jointly safe* if, for any other M', $TS(M) \geq TS(M')$ holds.

Blocking pairs: We let l denote the number of the preference layer in our model. That is, $l \in \{1, 2, 3\}$. We say that (p_j, d_i), which is in E but not in M, is a *blocking pair* for M on layer l iff p_j and d_i prefer each other to their current matches in M, say d_h and p_g, respectively: for $l = 1$, this means $\sigma_{ji} > \sigma_{gi}$ and $\sigma_{ji} > \sigma_{jh}$; for $l = 2$, this means $\pi_{ij} > \pi_{ig}$ and $\delta_{ji} < \delta_{jh}$; for $l = 3$, this means $d_i \succ d_h$ WRT ϕ_j and $p_j \succ p_g$ WRT χ_i.

Stable matchings: We say that matching M is *stable on layer* l iff, for each (p_j, d_i), that is in E but not in M, (p_j, d_i) is not a blocking pair for M on layer l. We say that matching M is *stable* iff, for each layer l, M is stable on l.

Efficient matchings: The total profit in M is $TP(M) = \sum_{(p_j,d_i)\in M\cap E} \pi_{ij}$. We say that matching M is *profit-efficient* if, for any other M', $TP(M) \geq TP(M')$ holds. The total delay in M is $TD(M) = \sum_{(p_j,d_i)\in M\cap E} \delta_{ji}$. We say that matching M is *delay-efficient* if, for any other M', $TD(M) \leq TD(M')$ holds.

4 Phase 1: Computing Jointly Safe Sub-Matchings

We warm up by considering how we might compute jointly safe sub-matchings over G. Many sexual harassment cases occur during rides in unsafe areas which drivers and passengers do not know very well, which increases the chance of victims submitting claims. From this perspective, we believe that focusing on achieving joint safety first is likely to reduce this chance because our notion of it is based on familiarity relations between passengers and drivers and the safety of areas.

4.1 Joint Safety

When computing jointly safe matchings, we need to respect the joint safety relation Σ and the eligibility relation E. For this purpose, we consider another bipartite graph $G_s = (V_P, V_D, E_s)$, where $E_s = \{(p_j, d_i) | (p_j, d_i) \in E, \sigma_{ji} = 1\}$. As it turns out, jointly safe matchings are maximum-cardinality matchings in the graph G_s and, therefore, can be computed in polynomial time.

Theorem 1. *There is an* $O(\max\{n, m\}^{5/2})$ *time algorithm that returns a jointly safe sub-matching.*

Proof. Given graph G_s, we can use the Hopcroft-Karp algorithm [6] for computing in $O(\max\{n, m\}^{5/2})$ time a maximum cardinality matching M_s over G_s. We argue that the returned matching is jointly safe. Indeed, if it were not, then there would be another M' such that $TS(M') > TP(M_s)$ would hold. Therefore, as each safety coefficient σ_{ji} is either one or zero, it would follow that M' would have a strictly greater cardinality than M_s. But, this would be in conflict with the correctness of the Hopcroft-Karp algorithm. □

4.2 Joint Safety and Efficiency

If the best safe match provides the worst profit then there is clearly a profit loss. If the best safe match provides the longest delay then there is clearly a delay loss. For this reason, we might wish to compute a profit-efficient or a delay-efficient matching from within the set of jointly safe matchings.

If we insist on achieving both types of efficiency, we may not be able to combine them with joint safety because there are instances where each of the matchings is neither profit-efficient nor delay-efficient. So, profit efficiency, and delay efficiency are not compatible in general. We demonstrate this Example 1.

Example 1. Let us consider $D = \{d_1, d_2\}$ and $P = \{p_1, p_2\}$. Furthermore, let us consider the only two possible matchings $M_1 = \{(p_1, d_1), (p_2, d_2)\}$ and $M_2 = \{(p_1, d_2), (p_2, d_1)\}$. We next define only the preferences on layer two as follows: the profits of driver 1 are $\pi_{11} = 2$ and $\pi_{12} = 1$; the profits of driver 2 are $\pi_{21} = 1$ and $\pi_{22} = 2$; the delays of passenger 1 are $\delta_{11} = 2$ and $\delta_{12} = 1$; the delays of passenger 2 are $\delta_{21} = 1$ and $\delta_{22} = 2$.

We note that M_1 is profit-efficient ($TP(M_1) = 4$) but gives a total delay of 4 ($TD(M_1) = 4$). By comparison, M_2 is delay-efficient ($TD(M_2) = 2$) but gives a total profit of 2 ($TP(M_2) = 2$). Hence, M_1 is not delay-efficient and M_2 is not profit-efficient. This means that neither M_1 nor M_2 satisfies both efficiencies.

By Example 1, it follows that it may not be possible to achieve joint safety, profit efficiency, *and* delay efficiency. Hence, we attempt to combine joint safety with either profit efficiency *or* delay efficiency. Such matchings always exist. To compute one of them, we designed Algorithm 1.

Algorithm 1 Joint safety and efficiency.

```
Input: P, D, E, Σ, Π, Δ
Output: a matching M_se that is jointly safe and efficient
Result: Joint safety on layer 1 and efficiency on layer 2
procedure JointSafety1Efficiency2GeneralCase
    G_s ← (V_P, V_D, E_s)
    M_s ← a jointly safe matching over G_s
    P_s ← the set of passengers matched in M_s
    D_s ← the set of drivers matched in M_s
    H_s ← (P_s, D_s, E_s)
    M_se ← an efficient matching over H_s      ▷ either profit-efficiency or delay-efficiency but both may not be guarateed
    return M_se
```

Theorem 2. *Algorithm 1 can return in* $O(\max\{n, m\}^3)$ *time a jointly safe and profit-efficient sub-matching, or a jointly safe and delay-efficient sub-matching, from within the set of jointly safe sub-matchings.*

Proof. Given G_s, Algorithm 1 can return, in $O(\max\{n, m\}^{5/2})$ time, a jointly safe sub-matching M_s: see Theorem 1. Next, it determines in $O(\min\{n, m\})$ time the graphs P_s and D_s, and also constructs in $O(\min\{n, m\}^2)$ time the graph H_s. Given H_s, Algorithm 1 can construct two weighted graphs, say W_{1s} and W_{2s}. $W_{1s} = (P_s, D_s, E_s, w_{1s})$ is such that, for each $e = (p_j, d_i) \in E_s$, $w_{1s}(e) = \pi_{ij} > 0$. $W_{2s} = (P_s, D_s, E_s, w_{2s})$ is such that, for each $e = (p_j, d_i) \in E_s$, $w_{2s}(e) = (K - \delta_{ji}) > 0$. It can then use a version [12] of the Hungarian algorithm [9] for computing in $O(\max\{n, m\}^3)$ time a maximum total weight (i.e. the sum of the individual edge weights) matching M_{se} over W_{1s} or W_{2s}. M_{se} is also a maximum cardinality matching over W_{1s} or W_{2s} as, otherwise, there would be another matching over W_{1s} or W_{2s} of strictly greater weight than M_{se} and this would contradict the correctness of the Hungarian method. Hence, M_{se} is also jointly safe over W_{1s} or W_{2s}. Finally, as M_{se} achieves the maximum total weight in W_{1s} or W_{2s}, it satisfies profit efficiency in W_{1s} or delay efficiency in W_{2s}. □

Finally, any jointly safe M_s over G_s is stable on layer one. Otherwise, there would be (p_j, d_i), that is in E but not in M_s, which would block two edges from M_s on layer one, say (p_j, d_h) and (p_g, d_i) because $\sigma_{ji} = 1$ but $\sigma_{jh} = 0$ and $\sigma_{gi} = 0$ would hold. But, then M_s would not be jointly safe over G_s.

5 Phase 2: Computing Stable Sub-Matchings

In Phase 1, we computed safe M_s over $G_s = (V_P, V_D, E_s)$. We let P_s denote the passenger set in M_s and D_s denote the driver set in M_s. For each $(p_j, d_i) \in V_P \times V_D$, that is not matched in Phase 1, it follows by the maximality of M_s in G_s that either $(p_j, d_i) \notin E$ or $\sigma_{ji} = 0$ holds. In this section, we consider another graph $G_e = (V_P \setminus P_s, V_D \setminus D_s, E_e)$, where $E_e = \{(p_j, d_i) | p_j \in V_P \setminus P_s, d_i \in V_D \setminus D_s, (p_j, d_i) \in E\}$. For each $(p_j, d_i) \in E_e$, we note that $\sigma_{ji} = 0$ holds because of $(p_j, d_i) \in E$ and $(p_j, d_i) \notin M_s$. Thus, we focus on preference layers two and three in our model.

If we want to maximize the total profit across matchings over G_e, then we can run the Hungarian algorithm with the underlying profit-weighted graph, as we did for graph H_s in Theorem 2. If we want to minimize the total delay across matchings over G_e, then we can run the Hungarian algorithm with the underlying delay-weighted graph, as we did for graph G_s in Theorem 2. By our running example, it follows that we may not be able to achieve profit efficiency and delay efficiency simultaneously. For these reasons, we investigate achieving stability on layers two and three simultaneously.

Stability often relates to reducing the risk of sexual harassment and its associated chance of submitting claims. For example, as most sexual harassment cases occur from male drivers to female passengers, we expect that female passengers rank female drivers as their top choice and male drivers rank female passengers as their top choice. However, we assumed previously that passengers do not submit their gender during the booking process. From this perspective, we believe that achieving stability with the gender preferences of passengers over drivers would limit the number of female passengers matched to male drivers.

If we consider just layer two or three, stable matching always exists because the corresponding preference relations in our model may contain ties but are complete [7]. But, matchings that are stable on both layers may not exist. This follows by the non-existence result for two-layer stable matchings from [3]. We designed therefore an experimental setup in which we wanted to test how close to two-layer stability we might get in practice. The architecture was MacBook Pro M1 2020, 16GB RAM, 250GB Disk Space, and macOS Ventura 13.0.

Instances For market size $n \in \{20, 40, 60, 80, 100\}$, we generated 10000 instances of n drivers who are eligible for n passengers. In each instance, there were two layers of preferences, one corresponding to layer two and another to layer three.

For layer two, we sampled uniformly at random each profit π_{ij} from $[1, 100]$, say in €, and each ETA t_{ij} from $[1, 60]$, say in *min*. In half of the instances, we set each departure time τ_j to 0, modeling *impatient* passenger behavior. This captured settings where passengers cannot wait before their departures (e.g. going from the airport to home). In the other half of the instances, we sampled uniformly at random each departure time τ_j from $(0, 30]$, modeling *patient* passenger behavior. This captured settings where passengers can wait before their departures or need some time to move to their pickup locations.

For layer three, we considered three gender types, namely 'male', 'female', and 'diverse', encoded as 0, 1, and 2, respectively. From among these gender types, we thus sampled uniformly at random the gender type of each p_j and each d_i. We also sampled uniformly at random the ranking over these gender types for each p_j and each d_i. By using these rankings, we calculated each ϕ_j and χ_i.

Algorithms We design an *eligibility* version of the Gale-Shapley algorithm [5] that uses strict passenger ordering. Thus, for each unmatched passenger p_j that comes next in the ordering, p_j makes a matching offer to the eligible driver they prefer, among the eligible drivers they have not yet already made an offer to. Each d_i, who has received an offer, evaluates it against their current match if they have one. If an eligible driver is not yet matched, or if they receive an offer from a passenger they prefer (based on their profit preferences or gender preferences which are known just to them) than their currently matched passenger, they accept the new offer and become matched to the new passenger. Otherwise, they reject the new offer. This process is repeated until matching is returned. With preferences on layer two (three), this returns a matching that is stable on layer two (three), but perhaps not stable on layer three (two). Thus, we propose the following extensions:

- RunGS2Count3: (1) run the eligibility version of the Gale-Shapley algorithm on layer two; (2) count how many pairs block the returned sub-matching on layer three, and
- RunGS3Count2: (1) run the eligibility version of the Gale-Shapley algorithm on layer three; (2) count how many pairs block the returned sub-matching on layer two.

5.1 Stability

RunGS2Count3 returns a matching that is stable on layer two but may not be stable on layer three. RunGS3Count2 returns a matching that is stable on layer three but may not be stable on layer two. For these reasons, we quantified the *stability gaps* of RunGS2Count3 and RunGS3Count2 on layers three and two by counting the numbers of pairs from these layers that block their returned matchings, respectively. The baseline is 0. It is achieved when no such blocking pairs exist.

Figure 2 shows our results. The gaps are bounded from above by the number n^2 of edges in the underlying graph. The gaps of RunGS2Count3 on layers two and three are 0 and around 20% of n^2, respectively. This supports its superior efficiency performance but may indicate a greater chance of harassment. The gaps of RunGS3Count2 on layers two and three are around 25% of n^2 and 0, respectively. This may indicate a lower chance of sexual harassment but supports its inferior efficiency performance.

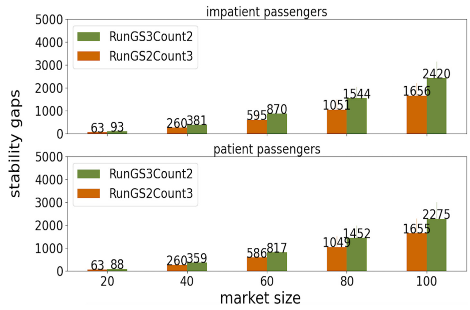

Fig. 2. The stability gaps of RunGS3Count2 and RunGS2Count3.

5.2 Stability and Efficiency

RunGS3Count2 and RunGS2Count3 induce efficiency losses. We used the minimum possible total delay—$\sum_{p_j \in P} \min_{d_i \in D} \delta_{ji}$—as a baseline for delay efficiency. Then, we calculated the *performance ratios for delay efficiency* between this baseline and the total delay of the matchings of RunGS3Count2 and RunGS2Count3. Similarly, we used the maximum possible total profit—$\sum_{d_i \in D} \max_{p_j \in P} \pi_{ij}$—as a baseline for profit efficiency. Then, we calculated the *performance ratios for profit efficiency* between the total profit of the matchings of RunGS3Count2 and RunGS2Count3 and this baseline.

Figure 3 depicts our results across all instances, where the lower-triangle trend traces the average performance of RunGS3Count2 and the upper-triangle trend traces the average performance of RunGS2Count3. The areas around these trends trace their worst and best performances, respectively.

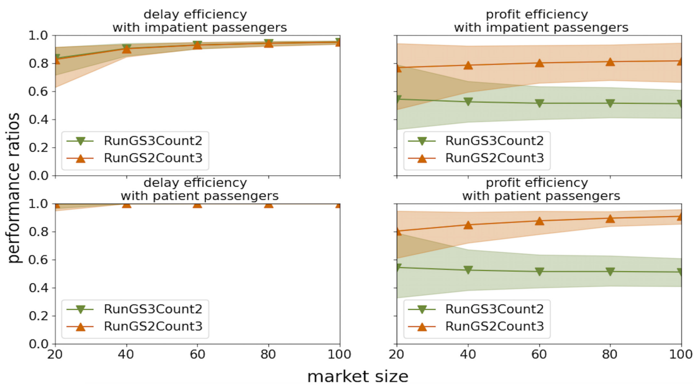

Fig. 3. The minimum, mean, and maximum performance ratios of RunGS3Count2 and RunGS2Count3. All ratios lie in $[0, 1]$.

Regarding delay efficiency, we made two observations (see the left plots in Fig. 3). Firstly, the performance of both algorithms converged steadily to optimality as we increased the market size. For example, with impatient passengers, their minimum performance ratios started from around 0.63 for $n = 20$ and increased to around 0.93 for $n = 100$. This could be because the greater the market size is the greater the expected number of most preferred options is. Secondly, the patience level of passengers mattered significantly as to how quickly the performance of both algorithms converged to optimality. For example, for $n = 20$, their minimum performance ratios started from around 0.63 with impatient passengers and increased to around 0.94 with patient passengers. This might be because the greater the departure time preferences are the lower the pickup delay preferences are.

Regarding profit efficiency, we also made two observations (see the right plots in Fig. 3). Firstly, as we increased the market size, the performance of RunGS2Count3 converged slowly to optimality, and the performance of RunGS3Count2 diverged slowly from optimality. For example, with impatient passengers, the minimum performance ratio of RunGS2Count3 increased from around 0.47 for $n = 20$ to around 0.66 for $n = 100$ and the maximum performance ratio of RunGS3Count2 decreased from around 0.79 for $n = 20$ to around 0.60 for $n = 100$. This is because RunGS2Count3 returns a matching that is stable on layer two whereas RunGS3Count2 is not guaranteed to return such a matching. Secondly, the patience level of passengers mattered significantly as to how steadily the performance of RunGS2Count3 converged to optimality. For example, for $n = 100$, its minimum performance ratio started from around 0.66 with impatient passengers and increased to around 0.85 with patient passengers. This might be because more patient passengers are indifferent among more drivers and, for this reason, such passengers could have picked more profitable drivers.

6 Conclusion and Future Work

We proposed an approach for possibly fighting sexual harassment during taxi rides by maximizing safety and optimizing stability when matching drivers and passengers. Our findings confirm that safe and stable matchings often exist. Thus, we lay down the blueprints for future work. For example, we will look at how we might deploy the disciplinary mechanism in our approach. Also, we will look at how we might generalize other single-layer properties to our setting. Finally, it will be interesting to quantify the probability of sexual harassment in taxi dispatching and derive theoretical bounds on how much achieving safety, stability, or efficiency decreases this probability.

Acknowledgements. Martin Aleksandrov was supported by the DFG Individual Research Grant on "Fairness and Efficiency in Emerging Vehicle Routing Problems" (497791398).

References

1. Breiding, M.J.: Prevalence and characteristics of sexual violence, stalking, and intimate partner violence victimization-national intimate partner and sexual violence survey, united states, 2011. Morbidity and mortality weekly report. In: Surveillance summaries (Washington, DC: 2002), vol. 63, no. 8, p. 1 (2014). https://ajph.aphapublications.org/doi/10.2105/AJPH.2015.302634
2. Brown, E.: Fare trade: reconciling public safety and gender discrimination in single-sex ridesharing. Yale L. & Pol'y Rev. **35**, 367 (2016). https://www.jstor.org/stable/26601904
3. Chen, J., Niedermeier, R., Skowron, P.: Stable marriage with multi-modal preferences. In: Proceedings of the 2018 ACM Conference on Economics and Computation, pp. 269–286. EC '18, Association for Computing Machinery, New York, NY, USA (2018). https://doi.org/10.1145/3219166.3219168
4. Fileborn, B., Cama, E., Young, A.: Perceptions of safety among taxi and rideshare service patrons: gender, safekeeping and responsibilisation. Int. J. Crime Justice Soc. Democr. **11**(4), 40 (2022). https://www.crimejusticejournal.com/article/view/2085
5. Gale, D., Shapley, L.S.: College admissions and the stability of marriage. Am. Math. Mon. **69**(1), 9–15 (1962). https://www.jstor.org/stable/2312726
6. Hopcroft, J.E., Karp, R.M.: A n5/2 algorithm for maximum matchings in bipartite. In: Proceedings of the 12th Annual Symposium on Switching and Automata Theory (Swat 1971), pp. 122–125. SWAT '71, IEEE Computer Society, USA (1971). https://doi.org/10.1109/SWAT.1971.1
7. Iwama, K., Manlove, D., Miyazaki, S., Morita, Y.: Stable marriage with incomplete lists and ties. In: Wiedermann, J., van Emde Boas, P., Nielsen, M. (eds.) Automata, Languages and Programming, pp. 443–452. Springer, Berlin Heidelberg, Heidelberg (1999). https://link.springer.com/chapter/10.1007/3-540-48523-6_41
8. Karp, R.M., Vazirani, U.V., Vazirani, V.V.: An optimal algorithm for on-line bipartite matching. In: Proceedings of the Twenty-Second Annual ACM Symposium on Theory of Computing, pp. 352–358. STOC '90, Association for Computing Machinery, New York, NY, USA (1990). https://doi.org/10.1145/100216.100262
9. Kuhn, H.W.: The Hungarian method for the assignment problem. Naval Res. Logist. Quart. **2**(1–2), 83–97 (1955). https://doi.org/10.1002/nav.3800020109. March
10. Lesmana, N.S., Zhang, X., Bei, X.: Balancing efficiency and fairness in on-demand ridesourcing. In: Wallach, H., Larochelle, H., Beygelzimer, A., d'Alché-Buc, F., Fox, E., Garnett, R. (eds.) Advances in Neural Information Processing Systems. vol. 32, pp. 5309–5319. Curran Associates, Inc. https://proceedings.neurips.cc/paper/2019/hash/3070e6addcd702cb58de5d7897bfdae1-Abstract.html
11. Mehta, A.: Online matching and ad allocation. Found. Trends Theor. Comput. Sci. **8**(4), 265–368 (2013). https://doi.org/10.1561/0400000057
12. Tomizawa, N.: On some techniques useful for solution of transportation network problems. Networks **1**, 173–194 (1971). https://onlinelibrary.wiley.com/doi/abs/10.1002/net.3230010206
13. Zhao, B., Xu, P., Shi, Y., Tong, Y., Zhou, Z., Zeng, Y.: Preference-aware task assignment in on-demand taxi dispatching: an online stable matching approach. In: Proceedings of the AAAI Conference on Artificial Intelligence, vol. 33, pp. 2245–2252 (2019). https://ojs.aaai.org/index.php/AAAI/article/view/4060