Summary

In this thesis we consider simplification algorithms for *radical expressions.* A radical over a field F is a root $\sqrt[d]{\rho}$ of some element ρ in the field F. A radical expression over F is any expression built from the usual arithmetic operations and from, possibly nested, roots.

In the first part of the thesis it is shown how to simplify an arbitrary sum of radicals over some real algebraic number field into a sum such that the radicals appearing in the transformed sum are linearly independent. In this form it is for example easy to check whether the original sum is zero. This can happen if and only if the coefficients in the sum resulting from the transformation step are zero.

The algorithms that transforms an arbitrary sum S of real radicals into a sum of linearly independent radicals is based on the following result due to C. L. Siegel.

Let F be any real field, i.e., $F \subseteq \mathbf{R}$. If for positive integers d_i and elements $\rho_i \in F$, $i = 1, \ldots, k$, $\sqrt[d_i]{\rho_i} \in \mathbf{R}$ and

$$\frac{\sqrt[d_i]{\rho_i}}{\sqrt[d_j]{\rho_j}} \notin F \text{ for all } i \neq j,$$

then $\sqrt[d_1]{\rho_1}, \ldots, \sqrt[d_k]{\rho_k}$ are linearly independent over F.

Hence the transformation can be done by determining for any pair of radicals in S whether its ratio is in F, by computing the representation of the ratio in F, and by finally collecting terms in order to determine the coefficients.

By generalizing the results above to sums of radicals that are not necessarily defined over the same field we naturally hit upon the problem of simplifying or *denesting nested radicals.*

Throughout the last years this problem has been studied intensively by various mathematicians and computer scientists. Many of them have been attracted by equations of Ramanujan such as the following:

$$\begin{aligned} \sqrt[3]{\sqrt[3]{2}-1} &= \sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}} \\ \sqrt[6]{7\sqrt[3]{20}-19} &= \sqrt[3]{\frac{5}{3}} - \sqrt[3]{\frac{2}{3}}. \end{aligned}$$

In this thesis we prove several structure theorems that explain Ramanujan's denestings. Based on these results we then describe algorithms that efficiently determine denestings such as the ones given by Ramanujan. We achieve the first denesting algorithms whose run time is polynomial in the output size.