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Version: 1

Differential Geometry II
Summer Semester 2024
Freie Universität Berlin

Exercise Sheet 8

Submission: 02.07.2024, 12:15 PM (start of lecture)

Note: This sheet contains 3 bonus points.

Exercise 1. (5 points)
Show that the set of *deck transformations* of a covering space E forms a group under composition of functions. Is this group commutative? Justify your solution.

Exercise 2. (3 points)
Let Γ denote the group of diffeomorphisms of a manifold M and M/Γ its quotient space consisting of equivalence classes $[p]$ where

$$p \sim q \iff \exists \Phi \in \Gamma : q = \Phi(p).$$

Show that $[p]$ is indeed an equivalence class.

Exercise 3. (8 points)
Show that $\pi : \mathbb{S}^1 \to \mathbb{S}^1$, $z \mapsto z^n$, $n \in \mathbb{N}$, is a covering of the unit circle (here, in the complex plane). Further, show that $\delta : \mathbb{S}^1 \to \mathbb{S}^1$, $z \mapsto ze^{\frac{2\pi i}{n}}$, is a deck transformation and that $\mathrm{deck}(\pi) \cong \mathbb{Z}/n\mathbb{Z}$.