Freie Universität Berlin
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WiSe 2024/25

Version: 1

Differential Geometry III – Homework 12

Submission: 31. January 2025, until 8:15 am (start of the exercise class).

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1. Exercise (4 points)

Calculate the homology $H_1(K)$ for the simplicial complex K shown in Figure 1.

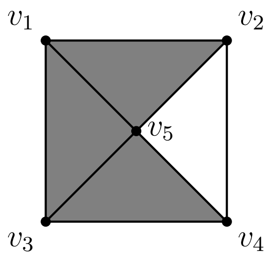

Figure 1: A simplicial complex K based on the set of abstract vertices $\mathscr{V} = \{v_1, \dots, v_5\}$.

2. Exercise (4 points)

Let K be a finite simplicial complex over an abstract set of vertices $\mathscr{V}$. Further, for $n \in \mathbb{N}$, let K_n be the simplicial complex

$$K_n := \{\{v_1\}, \dots, \{v_n\}, \emptyset\}$$

over the set of abstract vertices $\mathscr{V}_n = \{v_1, \dots, v_n\}$.

i) Calculate the homologies $H_p(K_n)$ for $p \in \mathbb{Z}$.[I]

ii) Prove that $H_0(K) \cong \bigoplus_{k=1}^{m} \mathbb{Z}$ for some $m \in \mathbb{N}$.[II]

iii) What is the topological interpretation of $\dim H_0(K)$ (i.e. the number m in ii))?

Total: 8

[I] Note, that $C_p(K) = \{0\}$ for integers $p < 0$ and that $\partial_p = 0$ for integers $p \leq 0$ (that means, ∂_p is the trivial homomorphism $c \mapsto 0$).

[II] In other words, prove, that there exist $m \in \mathbb{N}$ and $h_1, \dots, h_m \in H_0(K)$ such that every $h \in H_0(K)$ can be written as $h = \sum_{k=1}^{m} z_k h_k$ with unique $z_k \in \mathbb{Z}$.