Freie Universität Berlin WiSe 2024/25
Institut für Mathematik
Prof. Dr. K. Polthier, Dr. T. Kleiner Version: 1

Differential Geometry III – Homework 3

Submission: 15. November 2024, until 8:15 am (start of the exercise class).

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1. Exercise (2 points)

Show that the equator of the unit 3-sphere has mean curvature 0.

2. Exercise (3 points)

Calculate $\varphi_1, \varphi_2, \varphi_3$ and verify $\varphi_1^2 + \varphi_2^2 + \varphi_3^2 = 0$ for the Henneberg surface given by

$$\begin{aligned} f_1(u,v) &= 2\sinh(u)\cos(v) - \frac{2}{3}\sinh(3u)\cos(3v) \\ f_2(u,v) &= 2\sinh(u)\sin(v) + \frac{2}{3}\sinh(3u)\sin(3v) \\ f_3(u,v) &= 2\cosh(2u)\cos(2v). \end{aligned}$$

Note, φ_i, $i = 1, 2, 3$ are the components of the complexification[I] φ of f.

3. Exercise (3 points)

Determine the surfaces corresponding to the Weierstraß representations $F(z) = 2$, $G(z) = z$ and $F(z) = 2z^2$, $G(z) = z^{-1}$. What can you observe regarding these two surfaces?

Total: 8

[I] Cf. Wolfgang Kühnel - Differential Geometry, chapter 3D about minimal surfaces.